Department of Mathematics
Indian Institute of Technology Patna

MA 701 **ALGEBRA** **3-0-0-6**

Abstract Algebra:
Elementary set theory. Groups, subgroups, normal subgroups, homomorphisms, quotient groups, automorphisms, groups acting on sets, Sylow theorems and applications, finitely generated abelian groups. Examples: permutation groups, cyclic groups, dihedral groups, matrix groups. Basic properties of rings, units, ideals, homomorphisms, quotient rings, prime and maximal ideals, fields of fractions, Euclidean domains, principal ideal domains and unique factorization domains, polynomial rings. Elementary properties of finite field extensions and roots of polynomials, finite fields.

Linear Algebra:
Vector spaces, Bases and dimensions, Change of bases and change of coordinates, Sums and direct sums, Quotient spaces. Linear transformations, Representation of linear transformations by matrices, The rank and nullity theorem, Dual spaces, Transposes of linear transformations. Trace and determinant, Eigenvalues and eigenvectors, Invariant subspaces, Direct-Sum decomposition, Cyclic subspaces and Annihilators, The minimal polynomial, The Jordan canonical form. Spectral theorem for normal operators, Quadratic forms. Singular value decomposition, polar decomposition.

Text Books:

1. D. Dummit, R. Foote, *Abstract Algebra*, 3rd edition, Wiley, 2004.
2. K. Hoffman and R. Kunze, *Linear Algebra*, Prentice-Hall, 1996.

Reference Books:

1. S. Axler, *Linear Algebra Done Right*, 2nd Edition, UTM Series, Springer, 1997.
2. M. Artin, *Algebra*, Prentice Hall, 1994.

MA 711 **ANALYSIS-I** **3-0-0-6**

Topology:
Topological spaces, Basis for a topology, Limit points and closure of a set, Continuous and open maps, Homeomorphisms, Subspace topology, Product and quotient topology. Connected and locally connected spaces, Path connectedness, Components and path components, Compact and locally compact spaces, One point compactification. Countability axioms, Separation axioms, Urysohn's Lemma, Urysohn's metrization theorem, Tietze extension theorem, Tychonoff''s theorem, Completely Regular Spaces, Stone-Cech Compactification.

Functional Analysis:
Banach spaces, Continuity of linear maps, Hahn-Banach theorem, Open mapping and closed graph theorems, Uniform boundedness principle. Duals and Transposes. Compact operators and their spectra. Weak and Weak* convergence, Reflexivity. Hilbert spaces, Bounded operators on Hilbert spaces. Adjoint operators, Normal, Unitary, Self-adjoint operators and their spectra. Spectral theorem for compact self-adjoint operators, statement of spectral theorem for bounded self-adjoint operators.

Text Books:

1. James R. Munkres, *Topology*, 2nd Edition, Prentice Hall, 1999.
2. E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley, 1989.

Reference Books:

1. J. L. Kelley, *General Topology*, Springer International Edition, Indian Reprint, 2005.
2. M. A. Armstrong, *Basic Toplogy,* Springer- Verlag, 1997
3. M. Thamban Nair, *Functional Analysis: A First Course*, Prentice Hall of India, 2002.
4. G. F. Simmons, *Introduction to Topology and Modern Analysis*, Wiley, 2003.
5. J. B. Conway, *A Course in Functional Analysis*, GTM Series, Springer, 1990.

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MA 731 **DIFFERENTIAL EQUATIONS** **3-0-0-6**

Ordinary Differential Equations:
First Order ODE y'=f(x,y)-geometrical Interpretation of solution, Equations reducible to separable form, Exact Equations, Integrating factor, Linear Equations, Orthogonal trajectories, Picard's Theorem for IVP and Picard's iteration method, Euler' Method, Improved Euler's Method, Elementary types of equations. F(x,y,y') =0; not solved for derivative, Second Order Linear differential equations: fundamental system of solutions and general solution of homogeneous equation. Use of Known solution to find another, Existence and uniqueness of solution of IVP, Wronskian and general solution of non-homogeneous equations. Euler-Cauchy Equation, extensions of the results to higher order linear equations, Power Series Method application to Legendre Eqn., Legendre Polynomials, Frobenious Method, Bessel equation, Properties of Bessel functions, Sturm-Liouville BVPs, Orthogonal functions, Sturm comparison Theorem. Systems of Linear ODEs, Reduction of higher order linear ODEs to first order linear systems, Stability of linear systems.

Transforms:
Fourier Series, Fourier transform and Laplace Transform. Solving Differential Equations using Transform methods.

Partial Differential Equations:
Introduction to PDE, basic concepts, Linear and quasilinear first order PDE, Cauchy-Kowalewski theorem, second order PDE and classification of second order semilinear PDE (Canonical form), D' Alemberts formula and Duhamel's principle for one dimensional wave equation, Laplace's and Poisson's equations, Maximum principle with application, Fourier Method for IBV problem for wave and heat equation, rectangular region, Fourier method for Laplace's equation in three dimensions.

Text Books:

1. G. Birkhoff and G. C. Rota, *Ordinary Differential Equations*, 4th Edition, Wiley Singapore Edition, 2003.
2. I. N. Sneddon, *Elements of Partial Differential Equations*, Dover, 2006.

Reference Books:

1. E. A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, Tata McGraw Hill, 1984.
2. R. Haberman, *Applied Partial Differential Equations*, 4th Edition, Prentice Hall, 2003.
3. L. Perko, *Differential Equations and Dynamical Systems*, Springer – Verlag, 2006

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MA 712 **ANALYSIS-II** **3-0-0-6**

Complex Analysis:
Basic properties of the complex number system. Analytic functions, Cauchy-Riemann equations, elementary functions and their basic properties (rational functions, exponential function, logarithm function, trigonometric functions, roots functions). Cauchy's theorem and Cauchy's integral formula, Liouville's theorem, Morera's theorem and Maximum Modulus principle. Power series and Laurent series, isolation of zeros. Singularities and classification of isolated singularities (including singularity at infinity). Argument principle, Rouche's theorem, Casorati-Weierstrass Theorem, Schwarz's lemma, Residue theorem, evaluation of definite and improper integrals. Linear fractional transformations and mapping properties, Conformal maps. Statement of Riemann Mapping Theorem.

Measure Theory:
Algebras and sigma algebras, measures, outer measures, measurable sets, Lebesgue measure and its properties, non-measurable sets, measurable functions and their properties, Egoroff's theorem, Lusin's theorem; Lebesgue Integration: simple functions, integral of bounded functions over a set of finite measure, bounded convergence theorem, integral of nonnegative functions, Fatou's lemma, monotone convergence theorem, the general Lebesgue integral, Lebesgue convergence theorem, change of variable formula; Differentiation and integration: functions of bounded variation, differentiation of an integral, absolute continuity; Signed and complex measures, Radon-Nikodym theorem. Product measures, constructions, Fubini's theorem and its applications.

Text Books:

1. J. B. Conway, *Functions of One Complex Variable*, 2nd Edition, Springer, 2005
2.. G. de Barra, *Measure Theory and Integration*, New Age International, 2000.

Reference Books:

1. T. W. Gamelin, *Complex Analysis*, Springer, 2001.
2. D. L. Cohn, *Measure Theory*, 1st Edition, Birkhauser, 1994.
3. H. L. Royden, *Real Analysis*, 3rd Edition, Prentice Hall/Pearson Education, 1988.

MA 751 **ADVANCED OPTIMIZATION TECHNIQUES** **3-0-0-6**

Nonlinear programming: Convex sets and convex functions, their properties, convex programming problem, generalized convexity, Pseudo and Quasi convex functions, Invex functions and their properties, KKT conditions.
Goal Programming: Concept of Goal Programming, Model Formulation, Graphical solution method.
Separable programming.
Geometric programming: Problems with positive coefficients up to one degree of difficulty, Generalized method for the positive and negative coefficients.
Search Techniques: Direct search and gradient methods, Unimodal functions, Fibonacci method, Golden Section method, Method of steepest descent, Newton-Raphson method, Conjugate gradient methods.
Dynamic Programming: Deterministic and Probabilistic Dynamic Programming, Discrete and continuous dynamic programming, simple illustrations.
Multiobjective Programming: Efficient solutions, Domination cones.

Text Book:

1. Mokhtar S. Bazaaraa, Hanif D. Shirali and M.C.Shetty, *Nonlinear Programming, Theory and Algorithms,* John Wiley & Sons, New York (2004).

Reference Books:

1. D. G. Luenberger, *Linear and Nonlinear Programming*, Second Edition, Addison Wesley (2003).
2. R. E. Steuer, *Multi Criteria Optimization, Theory, Computation and Application*, John Wiley and Sons, New York (1986).

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MA 713 **TOPOLOGY** **3-0-0-6**

Quotient topology, Topological groups, Group Actions, Orbit spaces.

Homotopic maps, Construction of the fundamental group, Fundamental group of circle, Homotopy type, Covering spaces, Borsuk-Ulam and Ham-sandwich theorems, A lifting criterion, Seifert-van kampen theorem, Brouwer fixed point theorem and other applications

Polyhedra, PL maps, PL manifolds, Cell complexes, Subdivisions, Simplicial complexes, Simplicial maps, Triangulations, Derived subdivisions, Pseudomanifolds, Abstract simplicial complexes, isomorphism.

Orientation of complexes, Chains, Cycles and boundaries, Homology groups, Euler-Poincare formula, Barycentric subdivision, Simplicial approximation, Induced homomorphism, Degree and Lefschetz number fixed-point theorem.

Text Books:

1. C. A. Kosniowski, *First course in Algebraic Topology*, Cambridge Univ. Press, 2008.
2. C. P. Rourke, and B. J. Sanderson, *Introduction to Piecewise-Linear Topology*, Springer-Verlag, 1982.

Reference Books:

1. J. R. Munkres, *Elements of Algebraic Topology*, The Benjamin Cummings Pub., Co., 1984.
2. M. A. Armstrong, *Basic Topol*ogy, Springer (India), 2004.
3. K. Janich, *Topology*, Springer-Verlag (UTM), 1984.

MA 771 PROBABILITY THEORY AND STATISTICAL INFERENCE 3-0-0-6

Probability Theory: Algebra of sets, Measure and probability measure, Random variables, Standard discrete and continuous distributions, Conditional distributions and their independence, Distribution of functions of random variables, Expectation, Variance, Correlation, Moment generating functions and their properties, Convergence of random variables, Characteristic functions and properties, Laws of large numbers, Limit theorems.

Statistical Inference: Parametric and nonparametric models, Exponential families, Sufficiency, Completeness, Basu's Theorem, Invariance and maximal invariant statistic.

Point Estimation: Unbiased estimation, maximum likelihood estimation, method of moments, Loss functions, Risk Functions, Bayesian methods, Minimax and admissible estimators, Interval estimation, Equivariance Principle

Hypothesis Testing: Neyman-Pearson theory, Most Powerful (MP)Test, UMP Test, Unbiased Test, Monotone likelihood ratio property, Likelihood ratio tests, Wald's Sequential Probability ratio Test (SPRT), Invariant tests.

Text Books:

1. K.L. Chung, *A course in probability theory*, Second Edition, Academic Press, 2000
2. V.K., Rohatgi and Md. Ehsanes Saleh, *An introduction to probability and statistics*, Second Edition, Wiley India, 2009.

Reference Books:

1. G. Casella, and R.L. Berger, *Statistical inference*, Second Edition, Wadsworth, Belmont CA, 2001.
2. J.Shao, *Mathematical statistics*, Second Edition, Springer, 2008.
3. E.L.Lehmann and G.Casella, *Theory of point estimation*, Second Edition, Springer(India), 2003.

MA 752 MATHEMATICAL CONTROL THEORY 3-0-0-6

Linear Control Systems: Control systems and Mathematical modeling, classification of control systems, finite dimensional deterministic linear control systems, time variant and invariant control systems and their controllability, stability and stabilizability. **Nonlinear Control Systems:** Infinite dimensional spaces. State space form of an infinite dimensional linear and nonlinear control systems, controllability of similinear control systems: exact, approximate and null controllability. Second order systems and their exact and approximate controllability, semigroup theory and controllability questions for heat and wave equations. **Optimal Control systems:** optimal control problems for finite and infinite dimensional control systems including introduction to calculus of variations and maximum principle.

Reference Books:

1. Zabczyk Jerzy , *Mathematical Control Theory: An Introduction*, Series: Modern Birkhäuser Classics, 1st ed. 1992. 2nd, corr. printing 1995. Reprint, 2008.
2. R.G. Cameron and S. Barnett, *Introduction to Mathematical Control Theory*, Oxford Univ Press, 1990.
3. Eduardo D. Sontag, Mathematical Control Theory Deterministic Finite Dimensional Systems, Series: Texts in Applied Mathematics , Vol. 6, Springer, 1998.
4. A. Pazy, Semigroups of linear operators and applications to partial differential equations, Applied Mathematical Sciences, Vol. 44, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1983.
5. D. Subbaram Naidu, *Optimal Control Systems,* Series: Electrical Engineering Series Volume: 2, CRC Press, 2002.